Population games with attractiveness-driven, instantaneous and delayed strategy choice

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Attractiveness function and evolution equations

Evolution equations

$$\dot{p}_i = u(\frac{u_i}{u} - p_i)$$

where $u := \sum_{j=1}^{K} u_j$, $i = 1, 2, \ldots, K$.

Attractiveness function

$$u_i(t) = p_i^{1-\alpha} \nu_i^{1-\beta} \qquad i = 1, \ldots, K,$$

where $(\alpha, \beta) \in [0, 1] \times [0, 1]$.
For $\alpha = 1$, $u_i(p_i = 0) = 0$, $i = 1, 2$.

Sensitivity parameter

$$s = \frac{1-\beta}{\alpha}, \ \alpha \neq 0$$

Game with two strategies

$K = 2$

	C	D
C	R	S
D	T	P

where $R, S, T, P \geq 0$.

Evolution equation

$$\dot{p}_1 = p_1^{1-\alpha}\nu_1^{1-\beta} - p_1[p_1^{1-\alpha}\nu_1^{1-\beta} + (1-p_1)^{1-\alpha}\nu_2^{1-\beta}]$$

where ν_1 and ν_2 are average payoffs:

$$\nu_1 = Rp_1 + Sp_2 = Rp_1 + S(1-p_1) = (R-S)p_1 + S$$
$$\nu_2 = Tp_1 + Pp_2 = (T-P)p_1 + P$$

Game with three strategies (Rock-Paper-Scissors)

$K = 3$

	R	P	S
R	1	L	V
P	V	1	L
S	L	V	1

We assume that $V > 1 > L \geq 0$, where V, L – payoffs for victory and loss respectively.

Evolution equations

$$\begin{cases} \dot{p}_1 = p_1^{1-\alpha}\nu_1^{1-\beta} - p_1[p_1^{1-\alpha}\nu_1^{1-\beta} + p_2^{1-\alpha}\nu_2^{1-\beta} + \\ \qquad + (1-p_1-p_2)^{1-\alpha}\nu_3^{1-\beta}] \\ \dot{p}_2 = p_2^{1-\alpha}\nu_2^{1-\beta} - p_2[p_1^{1-\alpha}\nu_1^{1-\beta} + p_2^{1-\alpha}\nu_2^{1-\beta} + \\ \qquad + (1-p_1-p_2)^{1-\alpha}\nu_3^{1-\beta}] \end{cases}$$

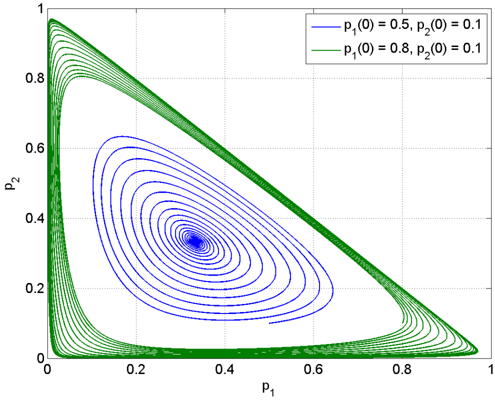

Figure: The trajectories of solutions to the evolution equations for a Rock-Paper-Scissors game with $V = 1.4$, $L = 0$ and $\alpha = 0.025$, $\beta = 0.825$.

Modifications of the attractiveness function

General attractiveness function

$$u_i = p_i^{1-\alpha}\nu_i^{1-\beta}f_i^{1-\beta}(1+k_i\frac{dp_i}{dt})(1+l_i v_i)^\gamma$$

where f_i, k_i, l_i, $\gamma \in \mathbb{R}^+$ are constant,
$v_i = \frac{D_i^2}{\nu_i^2}$ and D_i^2 is the variance of payoffs, $i = 1, \ldots, K$.

1. **transcendent factor** $f_i^{1-\beta}$
2. **rate factor** $(1+k_i\frac{dp_i}{dt})$
3. **selection potential** $(1+l_i v_i)^\gamma$

Nonconformist preferences

$$u_i = (1-p_i)^{1-\alpha}\nu_i^{1-\beta}$$

Iterated Prisoner's Dilemma

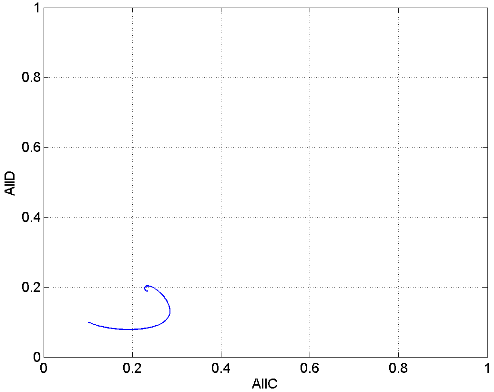

Figure: The trajectories of solutions to the evolution equations for an Iterated Prisoner's Dilemma with $f_1 = 1$, $\alpha = \beta = 0.1$, $[R, S, T, P] = [2, 0, 3, 1]$, $m = 10$, $f_2 = f_3 = 1$, $p_1(0) = p_2(0) = 0.1$.

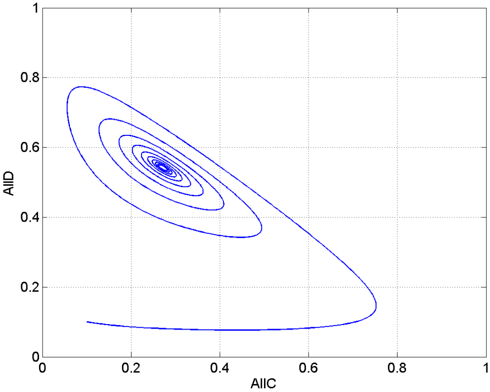

FIGURE: The trajectories of solutions to the evolution equations for an Iterated Prisoner's Dilemma with $f_1 = 1.6$, $\alpha = \beta = 0.1$, $[R, S, T, P] = [2, 0, 3, 1]$, $m = 10$, $f_2 = f_3 = 1$, $p_1(0) = p_2(0) = 0.1$.

Iterated Prisoner's Dilemma

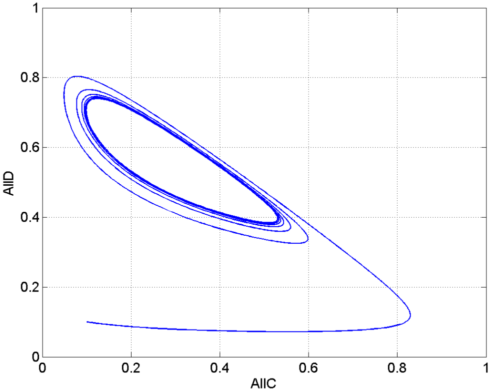

Figure: The trajectories of solutions to the evolution equations for an Iterated Prisoner's Dilemma with $f_1 = 1.75$, $\alpha = \beta = 0.1$, $[R, S, T, P] = [2, 0, 3, 1]$, $m = 10$, $f_2 = f_3 = 1$, $p_1(0) = p_2(0) = 0.1$.

Iterated Prisoner's Dilemma

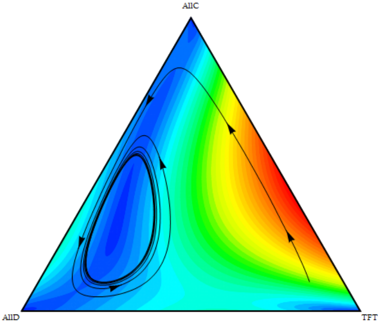

Figure: The trajectories of solutions to the evolution equations for an Iterated Prisoner's Dilemma with $f_1 = 1.75$, $\alpha = \beta = 0.1$, $[R, S, T, P] = [2, 0, 3, 1]$, $m = 10$, $f_2 = f_3 = 1$, $p_1(0) = p_2(0) = 0.1$.

Iterated Prisoner's Dilemma

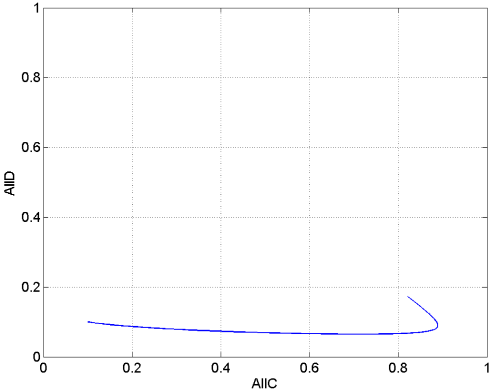

FIGURE: The trajectories of solutions to the evolution equations for an Iterated Prisoner's Dilemma with $f_1 = 1.9$, $\alpha = \beta = 0.1$, $[R, S, T, P] = [2, 0, 3, 1]$, $m = 10$, $f_2 = f_3 = 1$, $p_1(0) = p_2(0) = 0.1$.

Delays can be added to:

- payoffs

$$u_i(t) = (p_i(t))^{1-\alpha}(\nu_i(t-\tau))^{1-\beta}$$

- popularity

$$u_i(t) = (p_i(t-\tau))^{1-\alpha}(\nu_i(t))^{1-\beta}$$

- both popularity and payoffs

$$u_i(t) = (p_i(t-\tau_1))^{1-\alpha}(\nu_i(t-\tau_2))^{1-\beta}$$

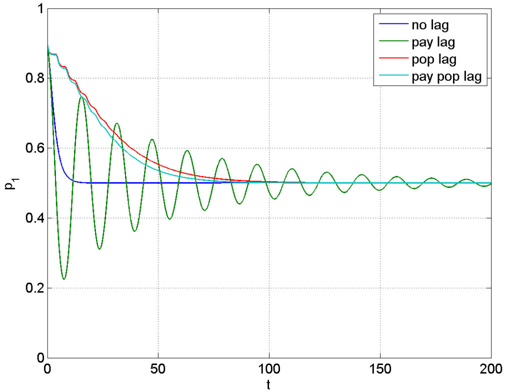

Figure: The trajectories of solutions to the evolution equation for a Snow-Drift game with $\tau = 4$, $[R, S, T, P] = [2, 1, 3, 0]$, $\alpha = 0.02$, $\beta = 0.2$.

Snow-drift game

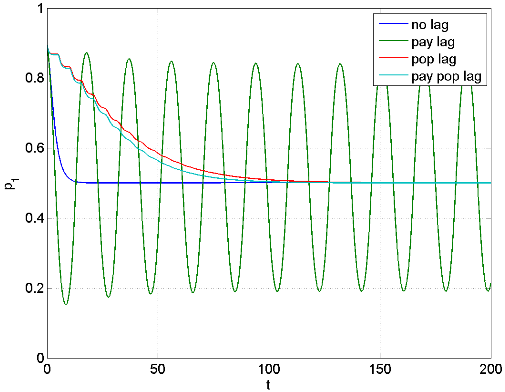

Figure: The trajectories of solutions to the evolution equation for a Snow-Drift game with $\tau = 5$, $[R, S, T, P] = [2, 1, 3, 0]$, $\alpha = 0.02$, $\beta = 0.2$.

Snow-drift game

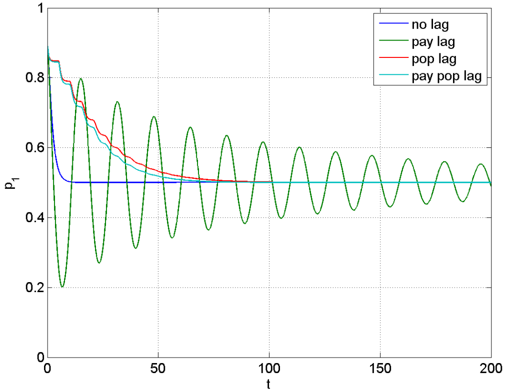

Figure: The trajectories of solutions to the evolution equation for a Snow-Drift game with $\tau = 5$, $[R, S, T, P] = [2, 1, 3, 0]$, $\alpha = 0.1$, $\beta = 0.2$.

Snow-drift game

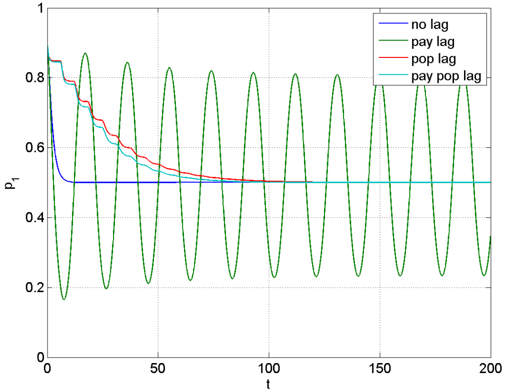

Figure: The trajectories of solutions to the evolution equation for a Snow-Drift game with $\tau = 6$, $[R, S, T, P] = [2, 1, 3, 0]$, $\alpha = 0.1$, $\beta = 0.2$.

Relation between sensitivity, delays and oscillations

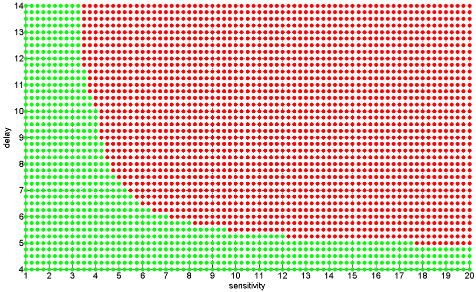

FIGURE: Graph presenting the relation between values of sensitivity and delays and the oscillatory behaviour of trajectories of solution to the evolution equations with attractiveness function delayed in payoffs for the Snow Drift game with payoff matrix $[R, S, T, P] = [2, 1, 3, 0]$. Green points denote damping oscillations and the red ones denote constant oscillations.

Rock-Paper-Scissors

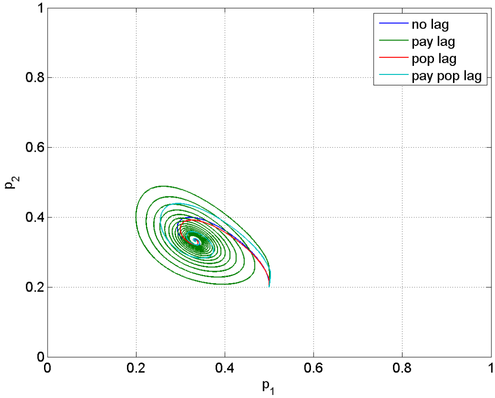

FIGURE: The trajectories of solutions to the evolution equations for a Rock-Paper-Scissors game with $\tau = 0.8$, $\alpha = 0.2$, $\beta = 0.2$, $V = 2$, $L = 0$, $p_1(0) = 0.5$, $p_2(0) = 0.2$.

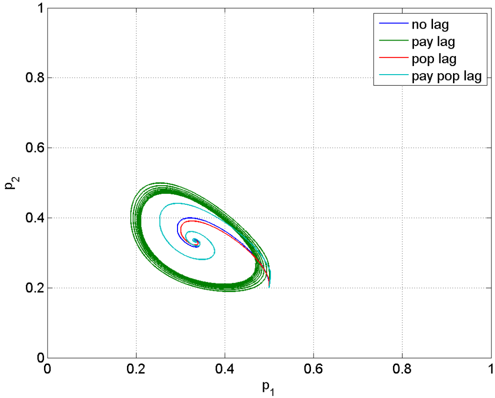

FIGURE: The trajectories of solutions to the evolution equations for a Rock-Paper-Scissors game with $\tau = 0.9$, $\alpha = 0.2$, $\beta = 0.2$, $V = 2$, $L = 0$, $p_1(0) = 0.5$, $p_2(0) = 0.2$.

Rock-Paper-Scissors

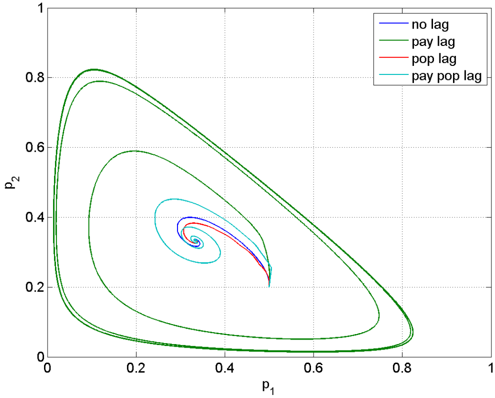

Figure: The trajectories of solutions to the evolution equations for a Rock-Paper-Scissors game with $\tau = 2$, $\alpha = 0.2$, $\beta = 0.2$, $V = 2$, $L = 0$, $p_1(0) = 0.5$, $p_2(0) = 0.2$.

Asymmetric games

	A	B
A	(a_1, a_2)	(b_1, b_2)
B	(c_1, c_2)	(d_1, d_2)

where $a_i,\ b_i,\ c_i,\ d_i > 0,\ i = 1, 2.$

Attractiveness function

$$u_A^i = x_i^{1-\alpha_i} \nu_{A_i}^{1-\beta_i} \qquad u_B^i = (1 - x_i)^{1-\alpha_i} \nu_{B_i}^{1-\beta_i}$$

Evolution equations

$$\begin{cases} \dot{x}_1 = (1 - x_1) x_1^{1-\alpha_1} \nu_{A_1}^{1-\beta_1} - x_1 (1 - x_1)^{1-\alpha_1} \nu_{B1}^{1-\beta_1} \\ \dot{x}_2 = (1 - x_2) x_2^{1-\alpha_2} \nu_{A_2}^{1-\beta_2} - x_2 (1 - x_2)^{1-\alpha_2} \nu_{B2}^{1-\beta_2} \end{cases}$$

where ν_{ji} is the mean payoff from strategy j in population i.

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